

# Dynamics of the giant planets of the solar system in the gaseous proto-planetary disk and relationship to the current orbital architecture

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## ABSTRACT

We study the orbital evolution of the 4 giant planets of our solar system in a gas disk. Our investigation extends the previous works by Masset and Snellgrove (2001) and Morbidelli and Crida (2007, MC07), which focussed on the dynamics of the Jupiter-Saturn system. The only systems that we found to reach a steady state are those in which the planets are locked in a quadruple mean motion resonance (i.e. each planet is in resonance with its neighbor). In total we found 6 such configurations. For the gas disk parameters found in MC07, these configurations are characterized by a negligible migration rate. After the disappearance of the gas, and in absence of planetesimals, only two of these six configurations (the least compact ones) are stable for a time of hundreds of millions of years or more. The others become unstable on a timescale of a few My. Our preliminary simulations show that, when a planetesimal disk is added beyond the orbit of the outermost planet, the planets can evolve from the most stable of these configurations to their current orbits in a fashion qualitatively similar to that described in Tsiganis et al. (2005).

*Subject headings:* planets and satellites: formation; solar system: formation

## 1. Introduction

Of all the planetary systems known to date, our solar system remains undoubtedly the one for which we have the largest number of accurate observational constraints to be used for modeling the evolution of the giant planets up to their current orbital configuration. Reconstructing this evolution as far back in time as possible is very important, because it can constrain the orbits on which the planets formed, and in turn, shed new light onto

their formation mechanism. In particular, knowledge of these aspects may allow us to understand why our solar system looks so different from the extra-solar systems discovered so far. Two differences are particularly striking. First, our giant planets are all far from the Sun, whereas giant planets in the very vicinity of their host stars are numerous in extra-solar systems. Observational biases favor the discovery of these planets, but the very fact that they exist in other systems and not in our own is real and remarkable. Thus, we need to understand in which conditions planets can avoid large-range radial migration towards the central star. Second, the orbital eccentricities of extra-solar planets, including those at distances of several AUs from the central star, are generally much larger than the eccentricities of the giant planets of our system. The latter, equal to several percent, are nevertheless non-negligible. Planet eccentricities are believed to be the result of mutual perturbations (Rasio and Ford, 1996; Marzari and Weidenschilling, 2002). It is important to understand which perturbations are responsible for the moderate eccentricities of the giant planets of the Solar System and why the orbital excitation could be much stronger in most extra-solar cases.

Our group has recently proposed two models that aim to reconstruct two different phases of the evolution of the solar system: the one that was dominated by the gas disk and the one that occurred after the disappearance of the gas.

The first model addresses specifically the migration of Jupiter and Saturn in the proto-planetary gas disk. If considered individually, these planets should have evolved towards the Sun, as the result of Type II migration. However, Masset and Snellgrove (2001, MS01 hereafter) showed that Saturn tends to get locked in the 2:3 MMR with Jupiter. In this configuration, the gaps opened in the disk by the two planets can overlap with each other. This can lead to a reversal of the migration direction. This mechanism has more recently been studied in greater details by Morbidelli and Crida (2007, MC07 hereafter).

They showed that, in the gas-disk parameter space represented by viscosity and scale height, there is a one-parameter family of solutions such that, once locked in the 2:3 MMR, Jupiter and Saturn do not migrate. Sets of parameters close to this family lead to inward or outward migration, but with rates that are much slower than the theoretically predicted Type II migration rates for a single planet. MC07 also showed that this kind of non-migrating, or slowly migrating, evolution is possible only if the planets involved have a mass ratio close to that of Jupiter and Saturn. Planets of similar masses, or - worse - systems where the outer planet is the more massive, inevitably lead to a fast inward migration. Therefore, they argued that the absence of a hot/warm Jupiter in our solar system is due to the specific mass hierarchy of our giant planets and to their formation on initially close-by orbits. To support this claim they pointed out that none of the known extra-solar planetary systems with two bodies close to their parent stars fulfill the conditions necessary to avoid Type II migration: either the planets have comparable masses, or the outermost one is the most massive, or they are too separated to have sculpted overlapping gaps in their primordial gas disks.

The second model (Tsiganis et al., 2005; Gomes et al., 2005), which actually was developed first, is often called the ‘Nice model’ because it was developed by an international collaboration at the Nice Observatory in France. This model argued that, if the giant planets had a more compact configuration at the end of the gas-disk phase, their subsequent migration driven by the interaction with a planetesimals disk could have forced them to cross some mutual mean motion resonance (MMR), thereby triggering a global instability; the current orbital configuration could then be achieved from the gravitational interaction between the planets and the disk particles. More precisely, the Nice model postulated that the ratio of the orbital periods of Saturn and Jupiter was initially slightly less than 2, so that the planets were close to their mutual 1:2 MMR; Uranus and Neptune were supposedly orbiting the Sun a few AUs beyond the gas giants, and a massive planetesimal disk was

extending from about 1.5 AU beyond the last planet up to 30–35 AU. As a consequence of the interaction of the planets with the planetesimal disk, the giant planets suffered orbital migration, which slowly increased their orbital separation. As shown in their N-body simulations, after a long quiescent phase (with a duration varying from 300 My to 1 Gy, depending on the exact initial conditions), Jupiter and Saturn were forced to cross their mutual 1:2 MMR. This event excited their orbital eccentricities to values similar to those presently observed. The acquisition of eccentricity by both gas giants destabilized Uranus and Neptune. Their orbits became very eccentric, so that they penetrated deep into the planetesimal disk. Thus, the planetesimal disk was dispersed, and the interaction between planets and planetesimals finally parked all four planets on orbits with separations, eccentricities and inclinations similar to what we currently observe. This model has a long list of successes. As already said, it explains the current orbital architecture of the giant planets (Tsiganis et al., 2005). It also explains the origin of the so-called Late Heavy Bombardment (LHB), a spike in the cratering history of the terrestrial planets that occurred  $\sim 650$  My after planet formation. In the Nice model, the LHB is triggered by the dispersion of the planetesimal disk; both the timing, the duration and the intensity of the LHB deduced from Lunar constraints are well reproduced by the model (Gomes et al., 2005). Furthermore, the Nice model also explains the capture of planetesimals around the Lagrangian points of Jupiter, with a total mass and orbital distribution consistent with the observed Jupiter Trojans (Morbidelli et al., 2005). More recently, it has been shown to provide a framework for understanding the capture and orbital distribution of the irregular satellites of Saturn, Uranus and Neptune (Nesvorny et al., 2007). The main properties of the Kuiper belt (the relic of the primitive trans-planetary planetesimal disk) have also been explained in the context of the Nice model (Levison et al., 2007; see Morbidelli et al., 2007, for a review).

The problem we are now facing is that a 2:3 MMR configuration of Jupiter and Saturn,

advocated to explain why the planets did not migrate towards the Sun, is different from the initial conditions of the Nice model, in which Jupiter and Saturn have initially non-resonant orbits,  $\sim 0.5$  AU interior to the 1:2 MMR. MC07 proposed a few mechanisms to extract Saturn from the 2:3 MMR with Jupiter and bring it close to the 1:2 MMR at the end of the gas disk phase. However, we believe that a more complete system (i.e. one with Uranus and Neptune) would probably become immediately unstable when this happens, in contrast with the Nice model.

This paper is the first in a series of two in which we explore this problem more thoroughly. In particular, we try to bridge the results of MS01/MC07 with the fundamental aspects of the Nice model. In section 2 we start from a configuration found in MC07 where Jupiter and Saturn are not migrating and we progressively add Uranus and Neptune into the problem. Following this procedure, we find 6 fully resonant relative configurations of the 4 planets, which are stable and avoid migration towards the Sun. In section 3, we study the stability of these configurations on longer timescales, after the gas disk has disappeared and in absence of a planetesimals disk. We find that 2 of the 6 configurations are stable over very long time (several  $10^8$  to  $10^9$  years). Although a detailed analysis of the evolution of these configurations under the influence of a planetesimal disk is left to the fore-coming paper, as a proof of concept, we present a couple of simulations in section 4, showing that the planets could eventually achieve an orbital architecture similar to their current one. The proposed evolution is different in the details from that of the Nice model (different resonances are involved in triggering the planetary instability) but the basic mechanism and evolution are the same. The conclusions of this paper follow, in the last section.

## 2. Four planet dynamics in the gas disk

We use the hydro-dynamical code developed by Crida et al. (2007) on the basis of the FARGO code by Masset (2000a,b), to simulate the dynamics of the planets in the proto-planetary gas disk. In the Crida et al. scheme, the disk is represented using a combination of 2-D and 1-D grids. The main part of the disk, in which the planets evolve, is sampled by a 2-D grid in polar barycentric coordinates. This grid extends from  $r = 0.3$  to 5 in radius (in units of the initial orbital radius of Jupiter) and has a resolution of 282 in radius and 325 in azimuth. The planets also evolve on coplanar orbits. The inner part of the disk (ranging from  $r = 0.016$  to the innermost boundary of the 2-D grid) and the outer part of the disk (ranging from the outermost boundary of the 2-D grid to  $r = 40$ ) are sampled by a 1-D grid. These 1-D grids have open outflow boundaries at  $r = 0.016$  and  $r = 40$ , while they exchange information with the 2-D grid at their common boundaries in order to supply realistic, time-dependent, boundary conditions for the latter. The algorithm for this interface between the 1-D and 2-D grids explicitly requires that the angular momentum of the global system (the disk in the 2-D section, plus the disk in the 1-D section plus the planets-star system) is conserved. With this approach, the global viscous evolution of the disk and the local planet-disk interactions are both described well and the feedback of one on the other is properly taken into account. Because the migration of the giant planets depends on the global evolution of the disk, this code provides more realistic results than the usual algorithms in which the evolution of the considered 2D portion of the disk depends crucially on the adopted (arbitrary) boundary conditions. For more information and accuracy tests we refer the reader to Crida et al. (2007).

We adopt a set of disk parameters from MC07 in which Jupiter and Saturn did not migrate after they became locked in the 2:3 MMR. The scale height of this disk is 5% and its viscosity (constant over radius) is  $\nu = 3.2 \times 10^{-6}$ , assuming the Sun’s mass and the initial

semi-major axis of Jupiter are the units of mass and distance, respectively. In the usual  $\alpha$  prescription (Shakura and Sunyaev, 1973), this viscosity corresponds to  $\alpha = 1.2 \times 10^{-3}$  at  $r = 1$ . The initial surface density of the disk is  $\Sigma(r) = 3 \times 10^{-4} \exp(-r^2/53)$ . This  $\Sigma$  was inspired by the results of Guillot and Hueso (2006) who studied the structure of a disk that viscously evolved under the effects of the collapse of fresh matter from the proto-stellar cloud, of the gas viscous spreading and of photo-evaporation by the central and neighboring stars. The exact choice of  $\Sigma(r)$  should not be crucial for our analysis, although some issues should be kept in mind.  $\Sigma$  is a multiplicative factor in the equations of motion, so it simply governs the evolution timescale. In case of differential migration of multiple planets, the value of  $\Sigma$  –here close to the minimal mass solar nebula (Hayashi, 1981)– determines the relative migration rates, and hence the probability of capture in the various mutual resonances. The radial profile of  $\Sigma(r)$  also affects the relative migration rates of planets at different locations. However, given that in our simulations (see below) all the planets are within a factor of 2.5 in heliocentric distance, the sensitivity of their relative evolution on the radial profile of  $\Sigma$  should be moderate. We will come back to these issue when discussing our results.

Fig. 1, reproduced from MC07, shows the evolution of the semi-major axes of Jupiter and Saturn, after they became locked in the 2:3 MMR, over 1,500 Jovian orbital periods. A slight parallel outward migration is visible, which could be annealed with a slight increase of the disk’s scale height or viscosity. The eccentricities have only small-amplitude oscillations around a small constant value (0.015 for Saturn and 0.004 for Jupiter). We refer to MS01 and MC07 for an explanation of why Type II migration is prevented in this configuration. We just stress here that this mechanism is robust. For a given (reasonable) viscosity, the coupled Jupiter-Saturn pair migrates outward in a thin disk, while it moves inward in a thick disk. Thus, it is always possible to find a disk scale height for which the migration vanishes. If some simulation parameters are changed (for instance the prescription of the



boundary conditions, the radial dependence of the viscosity, or the scale over which the potential of each planet is smoothed - here set to  $0.7 H$ , where  $H$  is the local height of the disk at the planet’s position - the exact value of the disk scale height that allows for a non-migrating solution may change, but the very existence of such a solution is not at risk.

The fact that Jupiter did not migrate closer to the Sun argues that the inward migration of the Jupiter-Saturn pair was, for the most part, inhibited. On the other hand, it is difficult to believe that Jupiter and Saturn migrated outward because the asteroid belt would have been completely decimated if Jupiter had been closer to the Sun. (Note that models that assume that Jupiter was, more or less, at its current location adequately reproduce the observed structure of the asteroid belt; see Petit et al., 2002, for a review). Thus, the philosophy of MC07, as well as of this work, is to construct models where the structure the proto-planetary disk is such that the planets do not migrate significantly. Given that this happens in our numerical scheme when  $H/r = 0.05$  and  $\nu = 3.2 \times 10^{-6}$  (for instance), we simply adopt these parameters with the understanding that the real disk may have been different.

We now extend the work of MC07 by adding Uranus and Neptune to the calculation. Given that the heliocentric order of the ice giants changed during  $\sim 50\%$  of the successful simulations of the original Nice model – so that we don’t know which one formed closer to the Sun – we assume that the two planets have the same mass: 15 Earth masses each. For simplicity, we nevertheless call the innermost ice giant ‘Uranus,’ and the outermost one ‘Neptune.’

The goal of this section is to find stable configurations for the four planets. To accomplish this we employ the following procedures. We pick up the MC07 simulation shown in Fig. 1 at  $t = 1,300$ , at which point we introduce Uranus into the calculation. In contrast to the procedures used by MC07 for Jupiter and Saturn, we let Uranus migrate

freely from the moment that it is introduced into the simulation. We think that this is a legitimate change because Uranus’s effect on the disk’s surface density profile is minimal and occurs on a timescale that is short compared to Uranus’s migration timescale.

In the first simulation, Uranus was initially placed at  $r = 2.55$ , which is beyond the 1:2 resonance with Saturn. Uranus moves inward relatively rapidly due to Type I migration. It jumps across the 1:2 resonance with Saturn (the resonance is too weak to capture it given the migration speed generated by our assumed disk; see also MS01) and is eventually trapped in the 2:3 resonance with Saturn (Fig. 2, black curve). In the second simulation, we start Uranus at  $r = 1.90$ , which is between the 2:3 and 1:2 resonances with Saturn. Again, the planet gets trapped in the 2:3 resonance (Fig. 2, dark gray curve). In both simulations, the capture into resonance increases the eccentricity of Uranus from  $\sim 0$  to  $\sim 0.025$ . After this capture, the three planets evolve in parallel, showing that they have reached a stable relative configuration in a three-body 2:3+2:3 resonance.

Notice that after this configuration is reached, Jupiter’s and Saturn’s outward migration is accelerated somewhat. We might expect the opposite since Uranus feels a negative torque from the disk that should be transmitted to Jupiter and Saturn through the resonances. The slow outward migration is due to the fact that Uranus slightly depletes the disk outside of Saturn’s orbit. As a consequence, the balance of the torques exerted on Jupiter and Saturn by the disk is broken, and the positive torque felt by Jupiter dominates. However, as we said above, this does not invalidate the general MS01/MC07 scenario because we could restore the torque equilibrium if we were to slightly increase in the scale height of the disk.

In a third simulation, we started Uranus at  $r = 1.73$ , which is between the 2:3 and 3:4 resonances with Saturn. Again, we observe an inward drift due to Type I migration until the planet is trapped in Saturn’s 3:4 MMR. After this, the relative configuration of the three planets does not change (see Fig. 2, gray curve), although, as described above,

the whole system migrates outward. The eccentricity of Uranus does not exceed 0.01. The evolution of Jupiter and Saturn are indistinguishable in all three simulations, and so we just plot those of the first simulation for clarity.

Furthermore, we perform a final simulation where we place Uranus initially at  $r = 1.61$ , which is between the 3:4 and 4:5 MMR with Saturn. In this case, the evolution is different from those described above. In particular, the motion of Uranus is unstable due to its proximity to the gas giants. Hence, Uranus is pushed outward until it finds again a stable relative configuration in the 3:4 MMR with Saturn (see Fig. 2, light gray curve). Its subsequent evolution is indistinguishable from the previous one.

From the above three experiments, we deduce that, for our assumed disk, there are two stable and invariant configurations of the three-planet system: Uranus being either in the 2:3 or the 3:4 MMR with Saturn.

Next, we introduce Neptune into the problem. We start by considering the first of the aforementioned simulations, where Uranus was trapped in the 2:3 MMR with Saturn. We continue this simulation after placing Neptune at  $r = 2.58$ , i.e. between the 2:3 and 1:2 MMR with Uranus. As expected, it drifts inward due to Type I migration (Fig. 3, black curve). However, Neptune is not trapped in Uranus’s 2:3 MMR, but crosses it, because the resonance is too weak to trap a body at Neptune’s migration speed. Neptune is, however, subsequently trapped in Uranus’s 3:4 MMR. Note that the 3:4 MMR with Uranus is also the 1:2 MMR with Saturn since Uranus and Saturn are in the 2:3 MMR. The capture of Neptune into resonance pumps the eccentricity of Uranus up to about 0.05, whereas the eccentricity of Neptune increases only to  $\sim 0.01$ .

Repeating the simulation with Neptune starting from  $r = 2.34$  (between the 3:4 and 2:3 MMR with Uranus), also leads to capture in the 3:4 MMR with Uranus. After the trapping, the evolution of the two systems are indistinguishable (Fig. 3, dark gray curve).

Conversely, starting the simulation with Neptune at  $r = 2.19$  or  $r = 2.11$ , leads to its capture into the 4:5 and 5:6 MMRs with Uranus, respectively (gray and light gray curves in Fig. 3). The eccentricities of the ice giants are progressively smaller with increasing  $m$ , for a  $m : m + 1$  resonance. For the 5:6 MMR, the eccentricity of Uranus and Neptune become  $\sim 0.04$  and  $\sim 0.007$ , respectively.

We repeat the same exercise, but this time in the system where Uranus was trapped in the 3:4 MMR with Saturn, and find similar results (see Fig. 4). The evolution of the eccentricities (not shown in the figure) is also similar to what is observed in the runs of Fig. 4. Thus, we conclude that for each of the two stable Jupiter-Saturn-Uranus configurations, there are three stable locations for Neptune: in Uranus’s 3:4, 4:5 or 5:6 MMR. Thus, we found 6 planetary configurations in total. In all cases, the four planets form a fully resonant system. In addition, all these configurations are characterized by an almost complete absence of radial migration, which is required to explain the absence of a hot/warm Jupiter in our solar system.

It is important to keep in mind that we cannot be sure that the 6 configurations found here are the only possible final states for the problem at hand. For example, we might reach a different configuration if we were to introduce Uranus and Neptune at the same time and on mutually scattering orbits in the vicinity of Jupiter or Saturn. Similarly, if we were to use a lower mass disk, the ice giants might be captured into weaker resonances – e.g. the 1:2 resonance with Saturn (for Uranus) or the 2:3 resonance with Uranus (for Neptune). However, these systems will be less compact than our six, and, given the results in section 4, we believe that they are unlikely to evolve into systems resembling the real giant planets. Thus, for the remainder of this paper we will restrict our analysis to the six configurations found in this section.

### 3. Evolution of the planets after the disappearance of the gas disk

In order to test the long-term stability of the planetary configurations constructed in the last section, we first have to transition from a gaseous to a gas-free environment. Gas disks are typically dispersed on a timescale of  $10^5$ – $10^6$  y (Haisch et al., 2001). Unfortunately, we do not yet know how this dispersal took place. Photo-evaporation is probably the key, but exactly how it proceeds (i.e. from the inside first, as argued in Alexander et al., 2006, or from the outside first, as found by Adams et al., 2004) it is still debated. Thus, we decided to implement a very simple transition in our hydro-dynamical code, where we do not change the shape of the disk’s surface density profile, but decrease the total amount of gas exponentially in time. We chose a decay rate such that the gas mass is halved in 160 “Jovian” orbital periods (i.e. at  $r = 1$ ). These hydro-dynamical are performed for 1,500 Jovian orbital periods, implying that the disk is reduced by a factor 670 at the end of the simulation, i.e. there is effectively no gas left.

Note that we are not claiming that the disappearance of the gas disk actually followed this simple recipe. Our aim is only to change the disk potential as smoothly and slowly as possible (given the available computing time) in order to give the planets enough time to adapt to the evolving situation. During these simulations we do not observe any significant evolution in the semi-major axes of the planets, and so the resonant structure is preserved. The eccentricities of some of the planets (particularly Uranus and Saturn) increase slightly, but attain new equilibrium values. The evolution of the eccentricities for the system with Saturn and Uranus in the 2:3 MMR and Uranus and Neptune in the 3:4 MMR is shown in Fig. 5.

Once the gas is removed, we can continue following the evolution of the our systems with an N-body code, accounting only for the Sun and the 4 planets. The simulations are done with the symplectic integrator SYMBA (Duncan et al., 1998) and cover a timescale of

1 Gy (assuming that  $r = 1$  corresponds to 5.1 AU so that the orbital period there is 11.5 y). The time-step is 0.2 y.

Note that because the hydro-dynamical simulations were carried out in two dimensions, the planetary orbits are strictly co-planar in our  $N$ -body simulations. We do not think that this is a significant limitation because it is well known that gas disks very effectively damp planetary inclinations (Lubow and Ogilvie, 2001; Tanaka and Ward, 2004) and we see no way to effectively excite them again. Planetary inclinations can only be excited by close encounters (which do not occur in our resonant configurations) or if the planets were trapped in inclination MMRs. Inclination resonances, however, are much weaker than the eccentricity resonances that occur at the same location (they act as second order resonances because the inclination has to appear with an even power in the equations of motion for D’Alembert rules — see Morbidelli 2002), so trapping in inclination resonances is highly unlikely. Thus, we expect that the real planets have small inclinations when they emerge from the gas disk, so that the study of the long-term stability of the multi-resonant configuration can be done effectively in two dimensions.

We find that the configuration with Saturn and Uranus in the 2:3 MMR and Uranus and Neptune in the 3:4 MMR remains stable for the full integration time, with no visible changes in semi-major axes or eccentricities (Fig. 6). Remember, however, that this simulation does not take into account the effects of a remnant planetesimal disk, which was used in the original Nice model to drive the planetary system unstable. We study this situation in section 4.

The configuration with Saturn and Uranus in the 2:3 MMR and Uranus and Neptune in the 4:5 MMR remains stable for 400 My. This time is long enough, however, that this configuration might be a reasonable starting point for a Nice-model-like evolution, because it is consistent with the 650 Myr delay between planet formation and the onset of the LHB.

All other configurations become unstable very quickly, at times ranging from a fraction of a My (for the most compact configuration) to 27 My (in the case with both Saturn and Uranus, and Uranus and Neptune in the 3:4 MMR). In these cases, however, we became concerned that we were driving these systems unstable because our disk dispersal time was too short. To test this possibility, we did again the later simulation with a gas-halving time of 1100 orbital periods at  $r = 1$ . The simulation was run for 10,000 orbital periods, at the end of which the gas surface density had been reduced by a factor of  $\sim 500$ . Even during the hydro-dynamical simulation we saw signs of instability and the system became unstable in less than 1 My in subsequent  $N$ -body simulation.

Another possibility is that the gravitational effects of a distant planetesimal disk could stabilize planetary configurations that were otherwise unstable. Thus, we redid each of the aforementioned  $N$ -body simulations twice: once adding a disk of 50 Earth-masses and once adding a 80 Earth-masses disk. In both cases, the disk was represented by a collection of 2,000 massive particles that ranged in heliocentric distance from just beyond the 2:3 MMR with the outermost planet, to  $\sim 30$  AU. We placed the inner edge of the disk this far from the Sun because we did not want a significant number of particles to leak out of the disk and trigger planetary migration. Such a migration would drastically change the structure of the system making a comparison with the disk-free simulations impossible. Such a distant disk could still irreversibly damp the planets' eccentricities through a secular exchange of angular momentum and mixing of the planetesimals' secular phases. Nevertheless, in all our simulations we found that the planetary configurations become unstable in very short periods of time ( $\sim 10$  My).

Given the above results, we believe that 4 out of the 6 relative configurations that we found are so unstable that they could not have lasted long enough to explain the 650 Myr delay between the formation of the planets and the LHB. This, however, does not preclude

the idea that other planetary systems might have passed through similar configurations — becoming unstable soon after the disappearance of the gas disk. Quite interestingly, we find that the instabilities that characterize these systems can often be much more violent than the one we can tolerate for the LHB, and involve close encounters between Jupiter and Saturn. As such, they can leave Jupiter on an orbit at about 4–5 AU with an eccentricity comparable to that of some extra-solar planets (see Fig. 7, also see Rasio and Ford, 1996, and Mazari and Weidenschilling, 2002). This opens the possibility that those extra-solar planets that have been found on eccentric orbits beyond  $\sim 3$  AU from their host stars might have followed an evolution similar to the ones that we found in our hydro-dynamical simulations, but that we rejected for our solar system, based on the LHB constraint.

Finally, we thought it would be instructive to investigate whether it was possible to add additional 15 Earth-mass ice giant to the system and still produce stable configurations. In order to have the best chance at successfully constructing such a system, we start with the most stable configuration produced above — namely the one in which Uranus is in the 2:3 MMR with Saturn and Neptune is in the 3:4 MMR with Uranus. We place a fifth planet, of equal mass to that of the other ice giants, between the 3:4 and 2:3 MMRs with Neptune at  $r = 2.87$ . We then let the system evolve under the effects of the gas disk. As expected, after a short period of Type I migration towards the Sun, the fifth planet is captured in the 3:4 MMR with Neptune. The capture into resonance excites the eccentricities of Uranus and Neptune, which stabilize at about 0.08 and 0.03 respectively. These values are about twice larger than those achieved in the simulations with only 4 planets, reported in Fig. 3 and 4. This is due to the fact that Uranus and Neptune have to retain the fifth planet from migrating through their mean motion resonances. Thus they suffer an additional negative torque. Because they are themselves locked in resonances, and therefore cannot migrate, this translates in a stronger excitation of their orbital eccentricities. On the other hand, the eccentricity of the fifth planet stays below 0.02.



After 2,000 Jovian orbital periods, we start the procedure of gradually depleting the gas disk using the procedure explained in the previous section. Unlike the previous cases, however, this system already shows signs of being unstable during this phase of its evolution. In particular, we see a secular increase in the amplitude of oscillation of both Jupiter’s and Saturn’s eccentricities. In addition, the variation in the eccentricities of all three ice giants become noticeably erratic. All this is probably a consequence of the enhanced eccentricities of Uranus and Neptune, relative to the 4-planets simulations.

Once the gas is gone, we move the system to our  $N$ -body code. We find that the system becomes violently unstable on a timescale of  $\sim 10$  My. To test this result, we have performed 3 additional  $N$ -body simulations starting from the output of the hydro-dynamical code at slightly different times, separated by 100 orbital periods at  $r = 1$ . In all cases the planets become unstable in less than 20 My. We have also performed runs where we have added either a 50 or 80 Earth-mass planetesimal disk (as described above). Again, the results are essentially the same. We only managed to slightly delay the onset of the instability to  $\sim 30$  My.

Of course, we cannot rule out that there was once a fifth fully grown planet in the outer solar system based on these simulations alone. After all, we only studied one configuration, and the fifth planet might have had a different mass or Uranus and Neptune might have been in different resonances than we assumed. Furthermore, as noted in the previous section, it is possible that other resonant configurations may have been reached if we changed the structure of the gas disk. Finally, there is a chance that the system was more unstable than it might have been because we dispersed the gas disk too quickly. Nevertheless, the striking difference between the behavior of our five-planet system and the four-planet case that has Uranus and Neptune in the same MMRs suggests that it is probably much more difficult to find a five-planet configuration that is stable for a long period of time.

#### 4. From a fully resonant evolution to the current orbital architecture

We conclude this paper by presenting a couple of ‘proof-of-concept’ N-body simulations in which the four giant planets interact with a trans-planetary disk of planetesimals that is close enough to the planets to cause them to migrate. The results demonstrate that the multi-resonant planetary systems described above can indeed evolve into an orbital configuration similar to that of the real giant planets.

We start with the most stable of our multi-resonant systems, namely the one in which Jupiter and Saturn are in the 2:3 MMR, Saturn and Uranus are in the 2:3 MMR, and Uranus and Neptune in the 3:4 MMR. As we have seen in Fig. 6, this system is stable for at least a billion years in the absence of external perturbations. We now add a trans-planetary disk of planetesimals. As in Tsiganis et al. (2005), we place the inner edge of the disk close to the outermost planet (0.5 AU beyond it), so that the planets migrate very quickly. This was a purely practical decision that allowed us to save a significant amount of computing time. As in Tsiganis et al. (2005), we model the disk with 1,000 equal-mass planetesimals, with a surface density profile that is inversely proportional to heliocentric distance,  $\Sigma(r) \sim 1/r$ . The outer edge of the disk is placed 30 AU, and its total mass is set to 50 Earth masses. All particles are initially on nearly circular and co-planar orbits with  $e \sim \sin(i) \sim 10^{-3}$ .

The initial conditions of the four planets are based on the output of the hydro-dynamical simulation with a decreasing gas disk. However, since our hydro-dynamical simulations were performed in two dimensions, they output only a co-planar configuration. This limitation is acceptable as long as the orbits of the planets do not cross one another. However, if they do cross (as we expect in these simulations) the 2D assumption artificially increases the chances of a collision to an unacceptable level. To avoid this technical problem, we add a small ( $\sim 10^{-3}$  AU/y)  $z$ -component to the velocity vector of each planet at the beginning of these  $N$ -body simulations. We followed the evolution of each system for 100 My using

SyMBA.

The result for our first run, shown in Fig. 8, has many of the characteristics seen in Fig. (1) of Tsiganis et al. (2005). In particular, the planets undergo a short period of smooth migration, during which they are on circular orbits. This is followed by an abrupt increase in the eccentricities of the gas giants, which destabilizes the orbits of the ice giants, and leads to a short, but violent, period of repeated encounters between the planets. Then there is a period when the planets migrate very quickly through the remaining disk, while their eccentricities slowly decay due to dynamical friction. The planets reach their final orbits in  $\sim 100$  My, when the planetesimals disk has been dispersed.

The essential ingredients of the Nice model are preserved in these new simulations. The initial orbits of the planets are stable and thus the instability does not occur until the planets are forced to migrate across a MMR. In the original Nice model the instability was caused by Jupiter and Saturn crossing the 1:2 MMR. Here, since Saturn starts off much closer to the Sun, the first important resonance that the planets encounter is the 3:5 MMR between Jupiter and Saturn. As Fig. 8 shows, this crossing causes the instability.

Using a perturbation theory similar to the one in the supplementary material of Tsiganis et al. (2005), one can show that the 3:5 MMR is less effective than the 1:2 MMR at increasing the eccentricities of the gas giants. However, since the planetary configuration that we used here is more compact than that in the original Nice model, a mild eccentricity “jump” is enough to destabilize the orbits of the ice giants. Thus, as we suggested in section 2, we can conclude that in order to reproduce a Nice-model-like instability in the orbits of the ice giants of a system that initially has Jupiter and Saturn locked in the 2:3 MMR, Uranus and Neptune probably need to be closer to the Sun than the original Nice model postulated.

In the run shown in Fig. 8, Uranus and Neptune suffer a few encounters with each

other before the latter is scattered into the disk (at about 15 AU). Dynamical friction from the disk decouples Neptune from Uranus, after which it migrates smoothly outward on a nearly circular orbit. Neptune stops migrating when it hits the outer edge of the disk. Notice that Uranus does not migrate far enough since its final semi-major axis is  $\sim 17$  AU instead of 19.2 AU. This behavior is reminiscent of the subset of simulations from Tsiganis et al. (2005), where the ice giants do not encounter Saturn. Indeed, the location of Uranus was one reason why Tsiganis et al. concluded that such encounters must have happened.

However, at the instability time, a number of different behaviors are possible due to the chaotic nature of the dynamics. For example, in a second simulation that is similar to the one in Fig. 8, but with a 65 Earth-mass disk, (Fig. 9) Saturn is involved in gravitational encounters with the ice giants. As a result, Neptune is thrown much farther into the disk, landing at  $a \sim 25$  AU on a very eccentric orbit. Its orbit is subsequently circularized by dynamical friction, and it comes to rest in a nearly circular orbit near 30 AU. In addition, Uranus’s final semi-major axis is very close to its observed value. This result is also consistent with the findings of Tsiganis et al. (2005). Notice that, in this run, the ice giant that formed closer to the Sun became the most distant planet in the final system.

As described above, in the original Nice model the orbital instability was caused by Jupiter and Saturn crossing the 1:2 MMR, while in these simulations it is caused by the 3:5 MMR. The 3:5 MMR is closer to the Sun than the 1:2 MMR, and, since Saturn is currently found beyond Jupiter’s 1:2 MMR, it eventually had to cross it. This resonance-crossing excites, once more, the gas giants’ eccentricities, and thus helps them maintain non-zero values against dynamical friction. We note that by the time Jupiter and Saturn cross their 1:2 MMR, the mass of the remaining disk in our runs is roughly 25 Earth-masses – a value that is well within the range needed to explain the capture of Jupiter Trojans during the 1:2 MMR crossing, according to the model in Morbidelli et al. (2005).

Of course, much more work is needed to build a successful ‘Nice model II’ that starts from an initial multi-resonant configuration of the giant planets. It is crucially important, for example, to determine whether it is possible to delay the instability for 650 Myr in order to be consistent with the LHB chronology. Recall that in the above simulations, we purposely set the disk’s initial distribution so that the resonance crossing occurs early in order to save CPU time. However, as discussed in Gomes et al. (2005), a more realistic distribution of the planetesimal disk should contain only particles whose dynamical lifetime is of order of the lifetime of the gas disk (few My) or longer. Assuming this disk distribution, Gomes et al. showed that, at least for the initial planetary configuration assumed in the original Nice model, the migration of the planets is slow enough that the instability is achieved only after hundreds of millions of years, consistent with the LHB timing. The same would hopefully happen for our new initial planetary configuration. Moreover, a large number of simulations need to be performed in order to quantify the probability that the final orbits achieved by the planets from our new initial configuration are consistent with observations. This study, which will require many time-consuming simulations is currently ongoing, and will be the subject of a forthcoming paper.

## 5. Concluding remarks

There are two important characteristics of our solar system that any model must be able to explain. First, the hot and warm Jupiters that are seen around some other stars are not present in our system. Second, the Moon and the planets most likely carry the scars of a spike in the impact flux that occurred  $\sim 650$  My (the Late Heavy Bombardment or LHB) after planets formed. The LHB strongly suggests that the planets suddenly became unstable at that time, destabilizing a massive reservoir of small bodies (Levison et al., 2001).

The so-called Nice model (Tsiganis et al., 2005; Gomes et al., 2005) has been proposed, in part, to explain the LHB. This model has a long list of successes in reproducing many of observational characteristics of the solar system. These include the number and the orbital distribution of the Jovian Trojans (Morbidelli et al., 2005) and the irregular satellites of Saturn, Uranus and Neptune (Nesvorný et al., 2007), and the structure of the Kuiper belt (Levison et al., 2007). The main weakness of the Nice model is that the initial conditions of the planets were chosen without concern for the previous phase of planetary evolution when the the proto-planetary gas disk was still in existence.

Recent hydro-dynamic simulations of Jupiter and Saturn embedded in a gas disk have supplied an important clue about the initial stable configuration of the planets. In particular, MS01 and MC07 showed that the absence of a hot/warm Jupiter in our system can be explained if these two planets had been locked in their mutual 2:3 MMR. The main goal of this paper, therefore, is to extended these results and find a four planet configuration that is both non-migrating while the gas disk is present, and dynamically stable long after the gas disk disperses.

To accomplish this goal, we have taken a system from MC07 consisting of Jupiter, Saturn, and a gaseous disk, and progressively added Uranus and Neptune, one at a time, into the simulations. We find that the interaction with the gas disk drives these planets into a configuration where each is in a MMR with its immediate neighbor(s). We have found six such “fully resonant” configurations, all of which are characterized by, at most, a small amount of radial migration. Four configurations, however, become rapidly unstable (on a timescale of a few My) after the disappearance of the gas disk. The remaining two were the least compact systems, with Saturn and Uranus in the 2:3 MMR and Uranus and Neptune in either the 4:5 or the 3:4 MMR. They were stable for 400 Myr and over a Gyr, respectively.

Furthermore, we have presented a pair ‘proof-of-concept’ simulations, showing that a quadruple resonant configuration, like the stable one above, can evolve into a system with a structure similar to that observed in the real solar system, once it interacts with a suitable trans-planetary planetesimal disk. The system evolves as follows. The migration of the giant planets, induced by the interaction with this disk, increases the ratios of the orbital periods between each pair of planets. Thus, the planets are extracted from their mutual, quadruple resonance. Because the system is very compact, new resonances are crossed during the migration. These resonance excite the eccentricities of the planets, triggering a global instability of the system. The orbits of the planets are eventually stabilized by the dynamical friction exerted by the planetesimal disk during its dispersal. Thus, this evolution is different from the original Nice model (Tsiganis et al. 2005) in the technical details only (e.g. different resonances are involved), but not in terms of the basic dynamical processes at work. More work is needed in order to quantify the statistical outcome of the chaotic evolution of the planets and, particularly, to prove that the onset of the planetary instability can occur late, as in Gomes et al. (2005). This will be the object of a forthcoming paper.

The long-term aim of this research is to build a bridge between our knowledge on solar system dynamics during the gas-disk era and that during the planetesimal-disk era, which remained up to now totally disconnected to each other. Success in this task would represent a significant advancement for our understanding of planet formation and of the key processes that made our solar system so different from all extra-solar systems discovered so far. In this paper we have taken the first steps toward this goal.

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Figure captions

Fig. 1 The evolution of Jupiter and Saturn in the gas disk ( $H/r = 0.05$ ,  $\nu = 3.2 \times 10^{-6}$ ) after they became locked in their mutual 2:3 MMR. From MC07.

Fig. 2 The evolution of Uranus, after it was added to the simulation presented in Fig. 1. Four simulations are presented. In the first (black line, labelled ‘Uranus sim.#1’), Uranus is started beyond the 1:2 MMR with Saturn. During its inward migration it passes across the 1:2 MMR and eventually gets trapped into the 2:3 MMR. In the second simulation (dark gray curve, labelled sim.#2), Uranus is introduced between the 2:3 and 1:2 MMRs with Saturn. Again, Uranus and gets captured into the 2:3 MMR. In the third simulation (gray line, labelled ‘Uranus sim.#3’), Uranus is started between the 3:4 and the 2:3 MMRs with Saturn and gets captured in the former. In the fourth simulation (light gray line, labelled ‘Uranus sim.#4’) Uranus is started between the 4:5 and 3:4 MMRs and it evolves outward until it is captured again in the 3:4 MMR. The evolution of Jupiter and Saturn is essentially the same in all the simulations, so we only present one example.

Fig. 3 The evolution of Neptune, after it was added to the simulation in which Uranus and Saturn are in the 2:3 MMR. Four simulations are presented. In the first one (black line, labelled ‘Neptune sim.#1’) Neptune is started beyond the Uranus 2:3 MMR, but jumps over it and gets captured in the Uranus 3:4 MMR. In the second one (dark gray line, labelled ‘Neptune sim.#2’), Neptune is started between the 3:4 and the 2:3 MMRs with Uranus and gets captured in the former. In the third simulation (gray line, labelled ‘Neptune sim.#3’), Neptune is started between the 4:5 and 3:4 MMRs and is captured in the former. In the fourth simulation (light gray line, labelled ‘Neptune sim.#4’), Neptune is started between the 5:6 and 4:5 MMRs and is captured in the former. The evolution of Jupiter, Saturn and Uranus is essentially the same in

the four simulations, so we only present one example.

Fig. 4 The same as Fig. 3, but for the case where Neptune is added to the simulation in which Uranus is in the 3:4 MMR with Saturn.

Fig. 5 The evolution of the eccentricities of Uranus (top, gray curve), Saturn (second from the top, dark gray curve), Neptune (third from the top, light gray curve) and Jupiter (bottom, black curve), during the simulation in which Uranus is in the 2:3 MMR with Saturn and Neptune is in the 3:4 MMR with Uranus. The surface density of the disk is now halved every 160 orbital periods at  $r = 1$ .

Fig. 6 The evolution of the giant planets over 1Gy, according to a N-body simulation, starting from the final output of Fig. 5. The top panel shows the evolution of the eccentricities with the same color code of Fig. 5. The bottom panel shows the evolution of the semi-major axes. The evolution looks perfectly regular and stable. No planetesimal disk is considered, and hence no migration of the planets relative to each other is observed.

Fig. 7 The semi-major axis vs. eccentricity distribution of the extra-solar planets discovered by radial velocity technique. The size of each dot is proportional to the planet’s radius (simply estimated from the cubic root of its  $M \sin(i)$ ). The rhombus shows the final orbit of Jupiter, achieved in the N-body simulation starting from the configuration with Uranus and Neptune both in the 3:4 MMR with the immediately interior planet. The excitation of Jupiter’s orbit is due to a strong encounter which ejects Saturn onto a very elongated orbit.

Fig. 8 (top) Evolution of  $a$ ,  $q$ , and  $Q$  for the outer planets, under the effects of a 50 Earth-masses planetesimals disk. For illustrative purposes, the unit of distance used in the hydro-dynamical simulations, is scaled such that Jupiter has initially a

semi-major axis of 5.42 AU. The unit of time is the year. The resonance crossing events are marked by vertical lines. The 3:5 MMR between Jupiter and Saturn is crossed at  $t \approx 6.5$  My, while the 1:2 MMR is crossed much later (at  $t \approx 63$  My). In this run Uranus falls  $\sim 2$  AU short of its true location. (middle) Eccentricity evolution for Jupiter and Saturn. The two excitation episodes are clearly seen, as is the slow damping due to dynamical friction. (bottom) A closer look at the instability-onset phase. The evolution of the semi-major axes of Saturn, Uranus and Neptune are shown in the interval 1 – 30 My. Also the evolution of the locations of the 3:5 MMR between Jupiter and Saturn and the 2:3 MMR between Uranus and Neptune are shown (time in log scale). Small variations in Uranus’  $e$  are visible, produced by the crossing of high-order resonances with Neptune. However, not even their mutual 2:3 crossing destabilizes their orbits. This occurs exactly after the crossing of the 3:5 MMR between Jupiter and Saturn, at  $t \approx 6.5$  My. After this event, Uranus and Neptune have repeated encounters and planetary migration is accelerated.

Fig. 9 The same as Fig. 8, for the run with a disk of 65 Earth masses (time in log scale). The crossing of the 3:5 MMR between Jupiter and Saturn occurs at  $t \approx 2$  My. Then, the innermost ice giant suffers close encounters with both Saturn and Jupiter, which receive a “kick” in  $a$  that forces them to cross their mutual 1:2 MMR. The eccentricity of the ice giant grows to  $\sim 0.6$ . Repeated encounters between the two ice giants follow, resulting to exchange the heliocentric ordering of their orbits. The planets are stabilized at 5.2, 9.2, 20 and 32 AU. The final eccentricities of Jupiter and Saturn are 0.03 and 0.07, respectively.

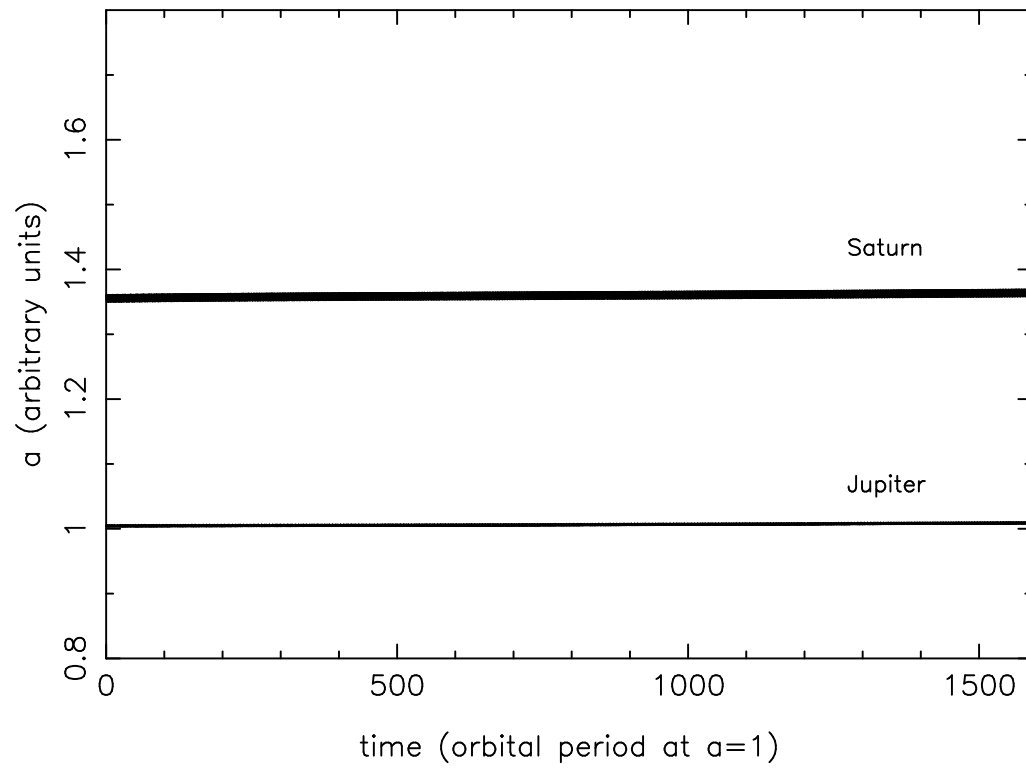


Fig. 1.—

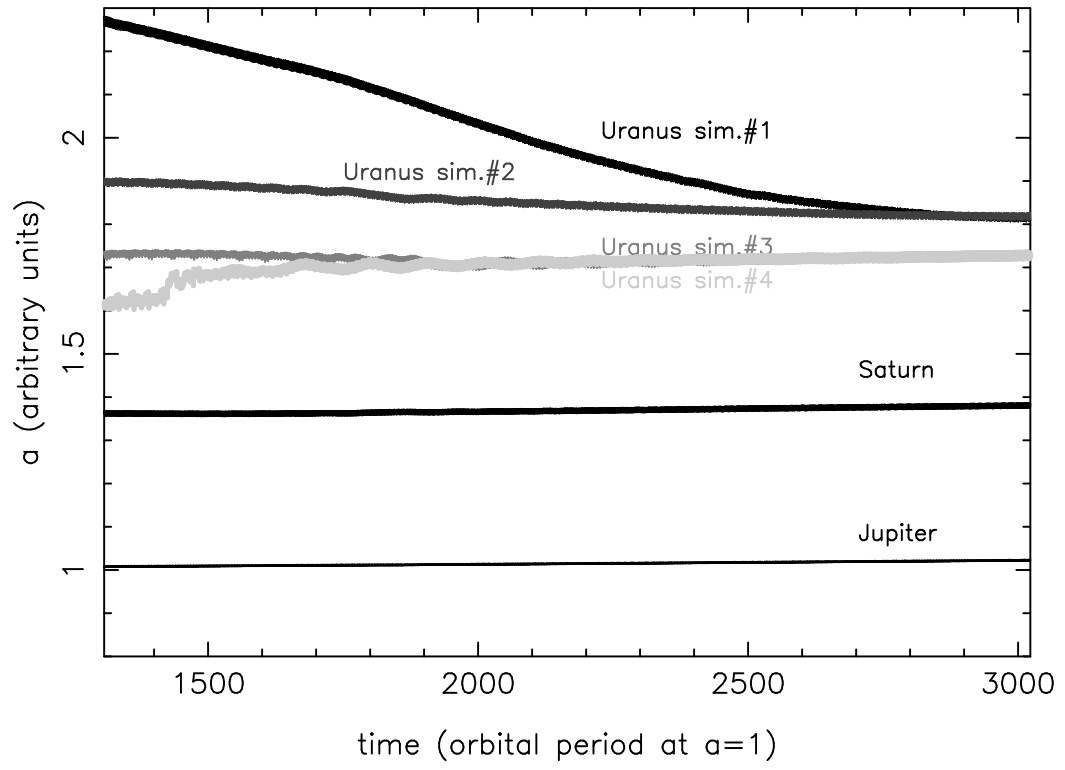


Fig. 2.—

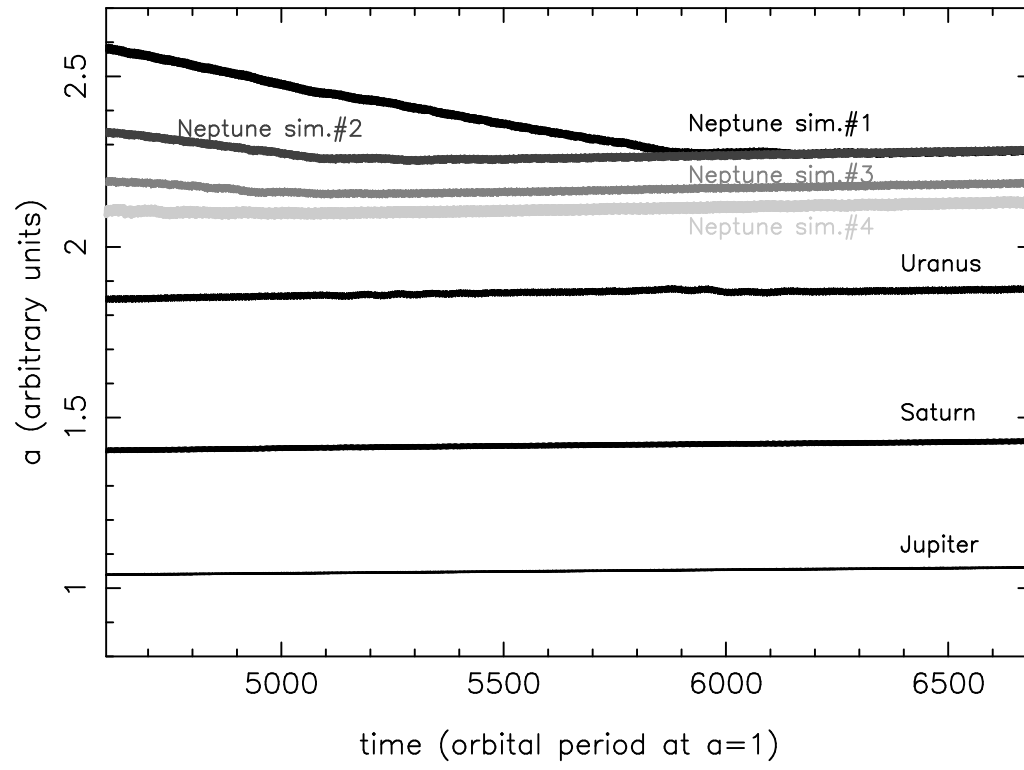


Fig. 3.—



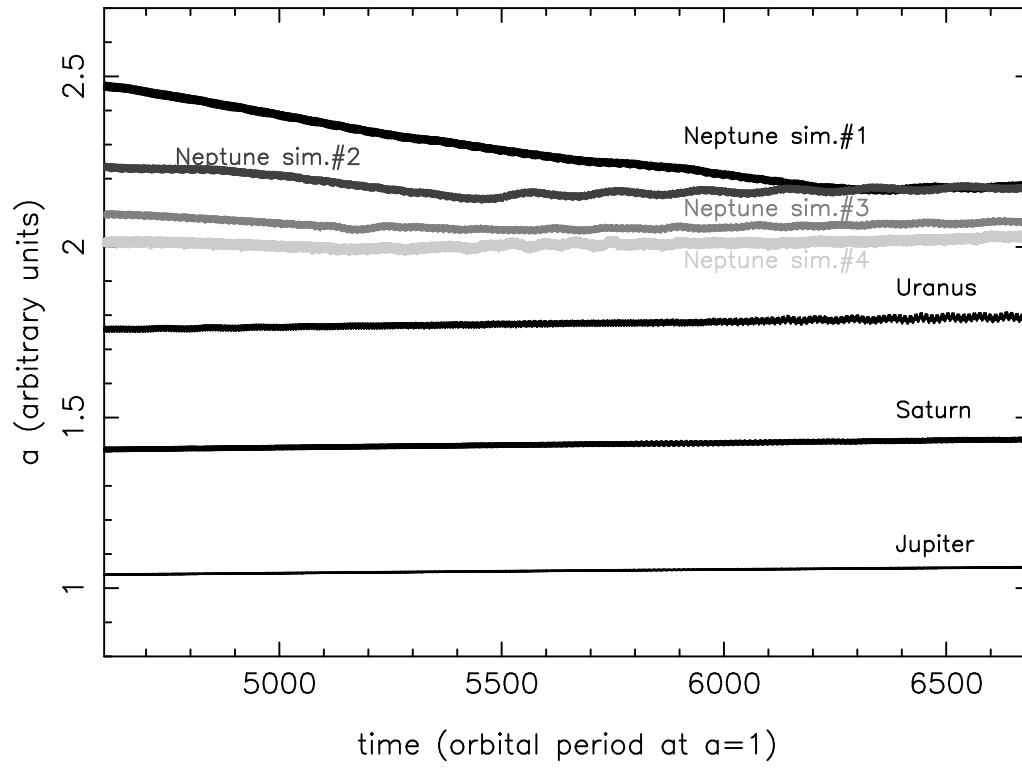


Fig. 4.—

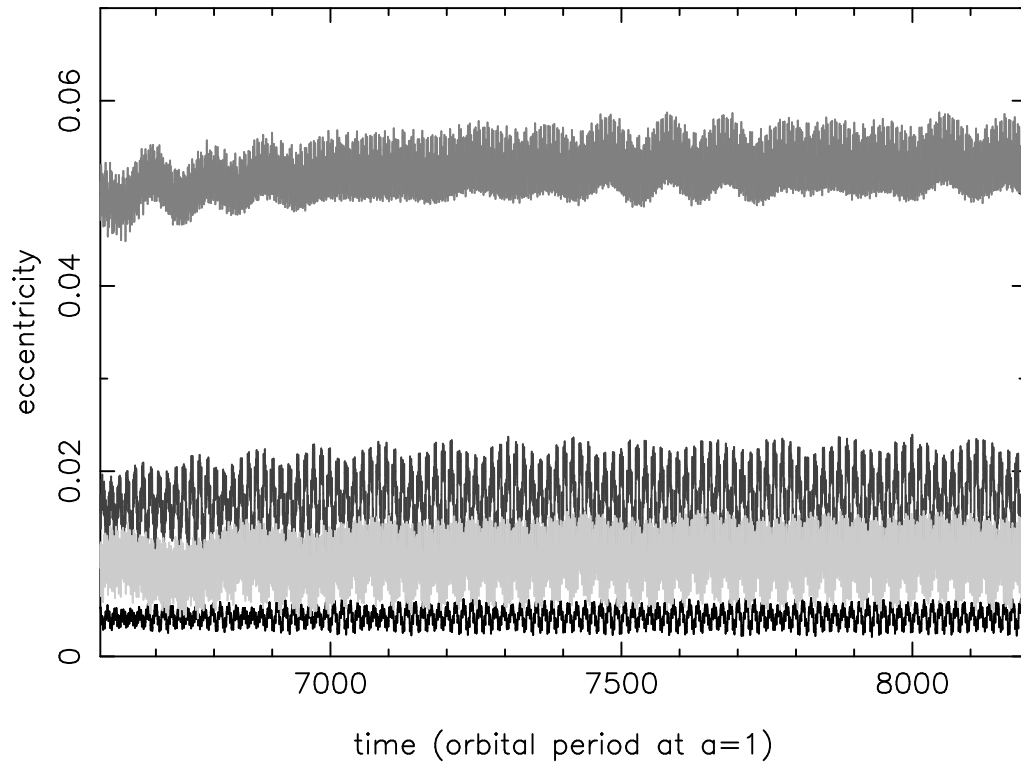


Fig. 5.—

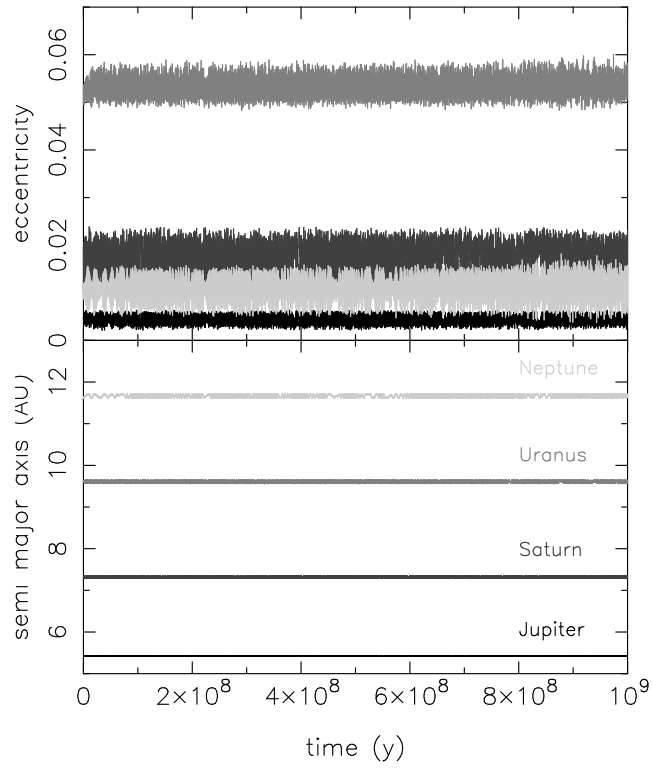


Fig. 6.—

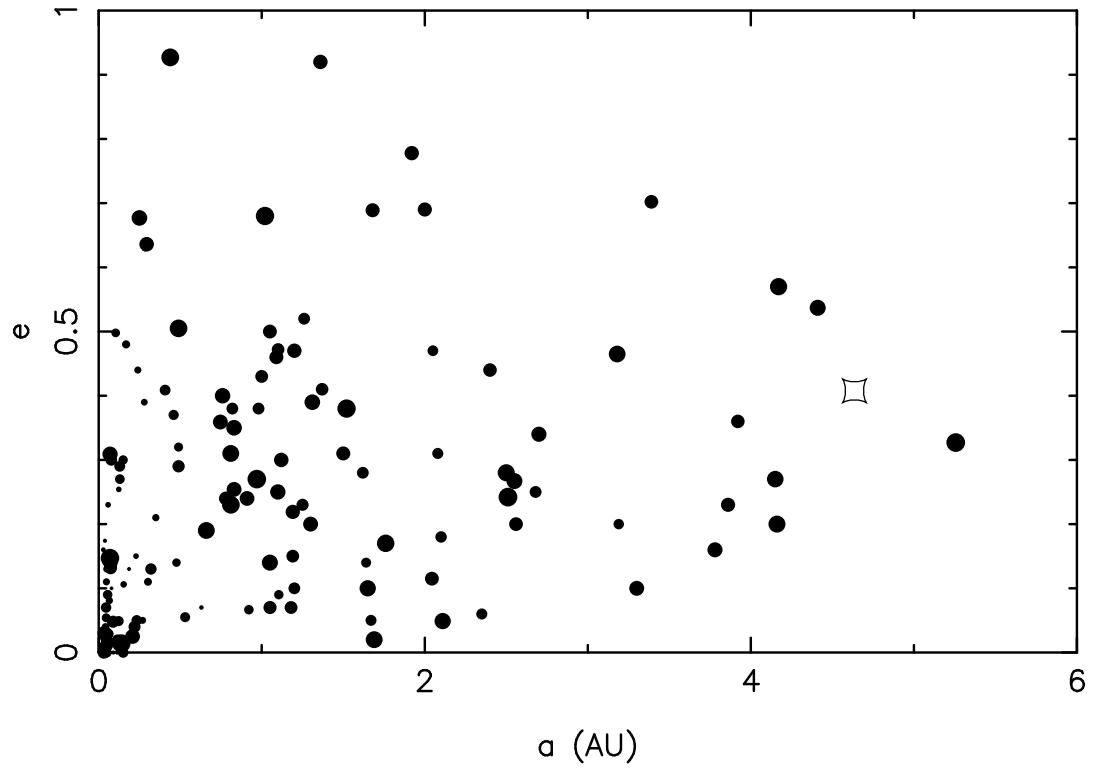


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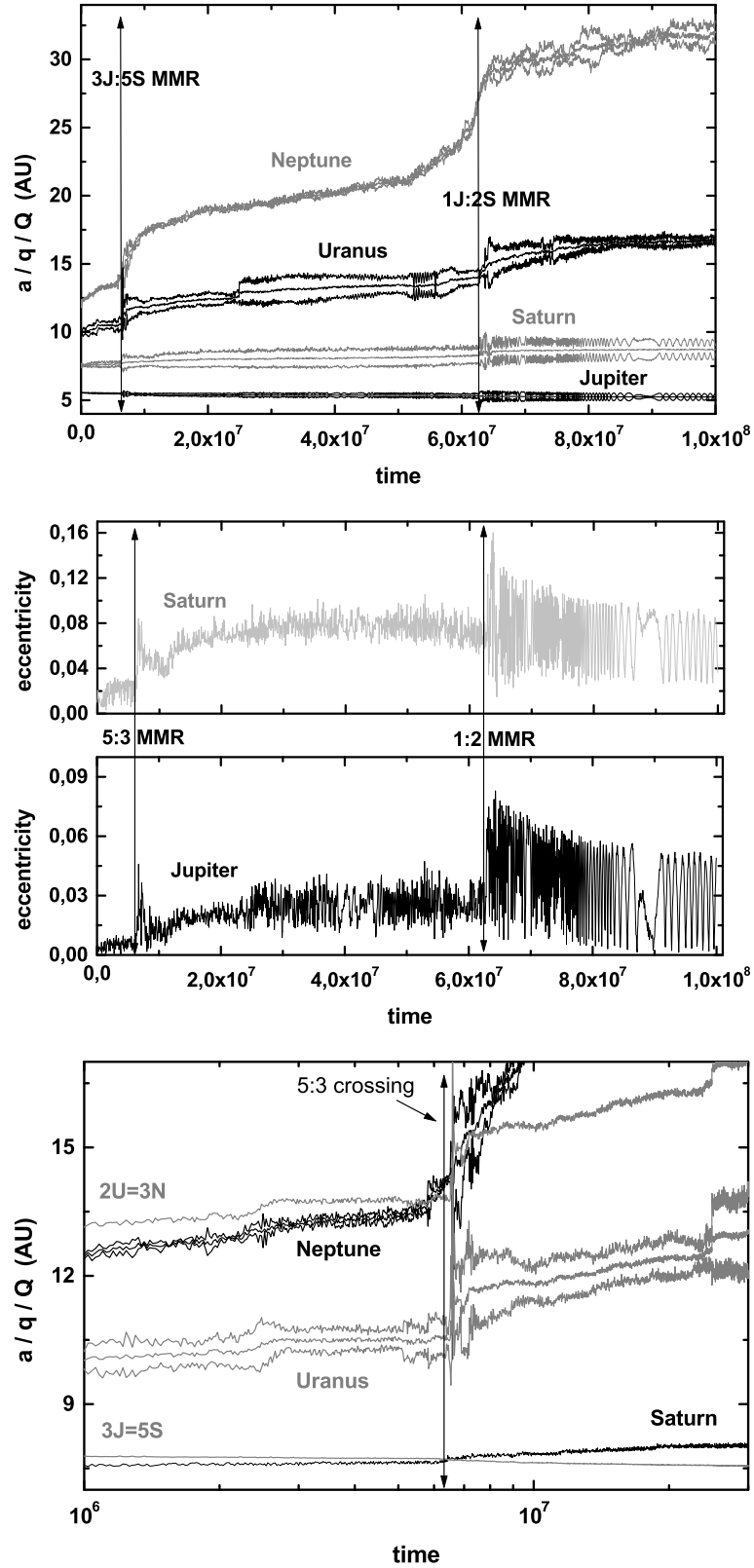


Fig. 8.—

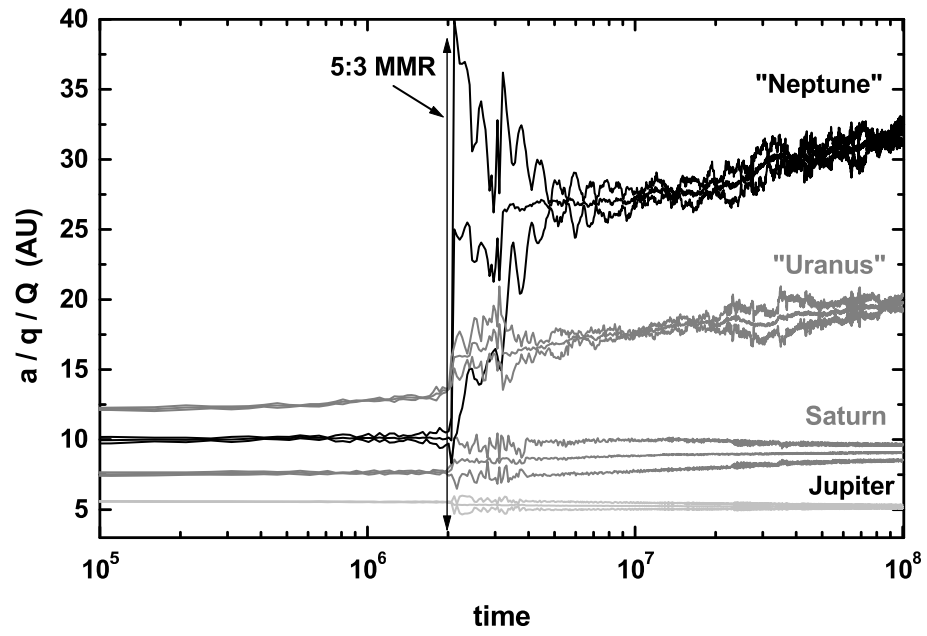


Fig. 9.—